Proceedings of the 37th Chinese Control Conference
July 25-27, 2018, Wuhan, China

Train Timetable and Trajectory Optimization Using Improved State-space MILP

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Abstract: Energy consumption in railway sectors is mainly dominated by train control and the timetable. This paper mainly discusses an energy-efficient train timetabling (EETT) method, which is under the total travel time limit and considers the energy-efficient train control (EETC) between consecutive stations. First, a non-linear EETC model is transformed into a mixed integer linear programming (MILP) model by discretization and linearization. Then, in order to improve the performance of MILP solver, especially the computational efficiency, a state-space improvement strategy is introduced, which could exclude those non-optimal state-spaces. Finally, two comparative simulations of HSRs are given. The results show that the proposed method not only can reduce the total energy consumption but also can reduce the computation time.

Key Words: Energy-efficient, Optimal train control, Timetabling, MILP, State-space

1 Introduction

With the development of rail traffic, it plays a more important role in the transportation of personnel and goods. Meanwhile, the huge energy consumption of rail traffic system becomes a tough challenge, due to the rising energy prices and environmental concerns [1]. Therefore, the interest of researches about energy-efficient train control and timetabling has been rising in recent years.

The optimal problem of EETC has been solved by various methods. Ichikawa [2] proposes a simplified train optimal control problem on level tracks, and proves the optimal control strategy consisting of maximum acceleration, cruising, coasting and maximum braking four driving regimes based on the Pontryagin maximum principle (PMP). The SCG group at the University of South Australia takes the PMP in into practice considering some realistic conditions *e.g.* speed limits, variable gradients and steep gradients. Howlett et al. [3]−[5] prove mathematically that the general optimal control strategy for tracks with varying speed limits and gradients. Howlett et al. [6] propose a local energy-efficient principle to calculate the switching points of control strategy on tracks with steep gradients. However, the method based on the PMP will not be a good choice when more realistic conditions are considered [7], thus numerical optimizations are adopted by many researchers. Vašak et al. [8] build a discrete-time model and approximate the nonlinear equations by a piecewise approximation (PWA) method. The optimal control strategy is pre-computed off-line with dynamic programming. Franke et al. [9] use kinetic energy instead of speed as a dynamic state variable and build a discrete-position nonlinear model, which is solved by dynamic programming. Ko et al. [7] propose a concept of the qualitatively non-optimal region to reduce the state-spaces, in order to reduce the computation time of the optimization process. Dynamic programming as a powerful optimization algorithm can be applied to actual complicated running condition of a train, except for terminal running time boundary condition. Wang et al. [10] build a linear discrete-position model and reformulate it to a MILP model using the PWA method and introducing logical and auxiliary variables. The MILP model can be solved efficiently by existing solvers, and the computation time is less than the one of other methods. Wang et al. linearize the varying maximum traction force using PWA method [11]. Meanwhile, Lu et al. utilize the MILP model to calculate the partial train optimal speed trajectory considering regenerative braking energy [12]. Although, the MILP model has good performance on computational efficiency, the rough discretization and PWA approximation cause bad effects to accuracy. In addition, heuristic algorithms have been applied to EETC problem due to the high computing power available nowadays. Ke et al. [13] propose a combinatorial optimization model to reduce the computation time, which is solved by ant colony optimization algorithms. Lechelle et al. [14] adopt a genetic algorithm to find energy-efficient control strategy and apply it to a tool called OptiDrive in engineering application. However, the heuristic algorithms cannot always find the optimal solution, and need lots of computation cost.

Based on the works on EETC problem between two stations, many researchers pay attentions to EETT problem on a railway corridor or network. Su et al. [15], [16] calculate the curves of running time and energy consumption to determine the optimal distribution of the running time supplements. Li et al. [17] propose a multi-objective train scheduling model by minimizing the energy cost and the total passenger-time, solved by a fuzzy multi-objective optimization algorithm. Yang et al. [18] adjust the headway and dwell time to maximize time overlaps of nearby accelerating and braking trains, in order to make the most of regenerative energy. Yang et al. [19] consider shortening the passenger waiting time into the model [18], solved by a genetic algorithm. Scheepmaker and Goverde [20] calculate the energy-efficient control strategy and timetable by

*This work is supported by the National Key Research and Development Program of China under Grant 2016YFB1200502, and the Fundamental Research Funds of Scientific and Technological Innovation Projects for the Central Universities of China under Grant 2682017CX048.

adjusting the coasting speed and cruising speed, based on PMP. Zhao et al. [21] adjust headway and running time to take full advantage of train regenerative braking energy, in order to minimize substation energy usage. Additionally, the aforementioned timetable researches are based on a simplified control strategy, adjusting the switching points of driving regimes or cruising speed. In this paper, we rebuild the optimal control problem to a MILP model based on the work [10], [11], and optimize the state-spaces to increase computational efficiency by several self-summarizing strategies. Taking advantage of the improved model, a running time distribution scheme and its corresponding concrete train control strategy are optimized synchronously.

This paper is organized as following: Section 2 introduces EETC problems between two stations and consecutive stations. Section 3 presents the procedures of reformulating the optimal train control problem to a MILP model. Section 4 introduces strategies to exclude non-optimal state-spaces. Section 5 provides two case studies between two stations and consecutive stations to illustrate the proposed method. Section 6 concludes this article.

2 Energy-efficient Train Control Problem

2.1 Dynamics Equation of Train Motion

The mass-point model of train is often used in researches about optimal train control problem. The dynamics equation of train motion can be formulated as the following simple continuous-position model [9]:

$$\begin{cases} m\rho \cdot v\dfrac{dv}{dx} = F(x) - B_E(x) - B_A(x) - W_0(v) - W_j(x) \\ \dfrac{dt}{dx} = \dfrac{1}{v} \end{cases} \tag{1}$$

where m represents the mass of train, ρ represents a factor to consider the rotating mass, v represents the train running speed, t represents the train running time, x represents the train running position, $F(x)$ represents the train traction force, $B_E(x)$ represents the train regenerative braking force, $B_A(x)$ represents the train air braking force, $W_0(v)$ represents the basic resistance, and $W_j(x)$ represents the line resistance.

In order to simplify the model, we introduce kinetic energy per mass unit $E = 0.5v^2$ [9]. Then, the continuous model (1) can be rewritten as:

$$\begin{cases} m\rho\dfrac{dE}{dx} = F(x) - B_E(x) - B_A(x) - W_0(\sqrt{2E}) - W_j(x) \\ \dfrac{dt}{dx} = \dfrac{1}{\sqrt{2E}} \end{cases} \tag{2}$$

Meanwhile, in order to avoid the generation of nonlinear equations, the basic resistance $W_0(v)$ is described as the Strahl formula [22]:

$$W_0(v) = m(a + bv^2) \tag{3}$$

where a and b represent coefficients described the basic resistance, which can be calculated based on lots of experimental data.

Then, the function (3) can be rewritten as:

$$W_0(v) = m(a + \frac{1}{2}bE) \tag{4}$$

2.2 Energy-efficient Train Control Model

The EETC model is built to reduce the operation energy consumption in a fixed running section under the total travel time limit. The running speed must be lower than the limited speed. The traction and braking force, as control variables, are bounded by the maximum traction force and the maximum braking force, which are influenced by the speed and train characteristics. Considering the utilization of regenerative braking. the operation energy consists of the traction energy consumption and the regenerative braking energy feedback, thus the objective function can be described as:

$$\min\ J_r = \int_{x_{\text{start}}}^{x_{\text{end}}} \left(\frac{F(x)}{\eta} - \alpha B_E(x) \right) dx \tag{5}$$

subject to the dynamics equation of train motion (2) and the following constrains [11]:

$$\begin{cases} 0 \le F(x) \le F_{\max}(\sqrt{2E(x)}) \\ 0 \le B_E(x) \le B_{E,\max}(\sqrt{2E(x)}) \\ 0 \le B_A(x) \le B_{A,\max}(\sqrt{2E(x)}) \\ 0 \le E(x) \le E_{\max}(x) \\ E(x_{\text{start}}) = 0 \quad t(x_{\text{start}}) = 0 \\ E(x_{\text{end}}) = 0 \quad t(x_{\text{end}}) = T \end{cases} \tag{6}$$

where J_r represents the operation energy consumption, x_{start} represents the start position of the running section, x_{end} represents the end position of the running section, η represents the efficiency of motor, α represents the efficiency of generator, $F_{\max}$ represents the maximum traction force, $B_{E,\max}$ represents the maximum regenerative braking force, $B_{A,\max}$ represents the maximum air braking force, T represents the total travel time, which is determined by the timetable. $E_{\max} = 0.5V^2$ and V represents the limited speed.

In this paper, we consider the neutral section constraint, where the control strategy is coasting, which can be described as:

$$F(x_{\text{ns}}) = B_E(x_{\text{ns}}) = B_A(x_{\text{ns}}) = 0 \tag{7}$$

where x_{ns} represents the position of neutral section.

In this continuous-position model, the kinetic energy per mass unit $E = 0.5v^2$ and the running time t are state variable, the running position x is the independent variable.

2.3 Multiple Running Sections Model

The EETC model formulated above can be used to solve the optimal control problem between two sections barely. Then, we build a model to describe the problem of EETT and its corresponding control strategy between consecutive stations. In regard to a running path consists of K stations (one originating station, one terminal station, $K-2$ middle stations), the objective function can be described as:

$$\min\ J_r = \sum_{i=1}^{K-1} \int_{S_i}^{S_{i+1}} \left(\frac{F(x)}{\eta} - \alpha B_E(x) \right) dx \tag{8}$$

subject to the constrains (2), (6), (7) and the scheduled stop constrains as following, which are determined by timetable.

$$E(S_i) = 0,\ i = 1, 2, \ldots, K \tag{9}$$

where S_i represents the position of station(i), $i = 1,2,\ldots,K$. Note that $S_1 = x_{\text{start}}$ and $S_K = x_{\text{end}}$.

The running time of each section is unlimited, because we just impose constrains on the total travel time from station S_1 to S_K. Thus, the distribution scheme of the total running time of each section is optimized when the energy-efficient train control strategy is determined.

3 The MILP Approach

3.1 The Discrete-position Model

As proposed in some previous works [9], [10], it is feasible to split the running path [x_{start}, x_{end}] into N intervals to build a discrete-position model. Meanwhile, the line resistance, train traction and braking force are assumed to be constant in each interval [x_k, x_{k+1}] with uniform length. However, the method to split the running path into uniform length intervals poses a challenge to show the characteristics of line resistance $W_j(x)$ and limited speed V concretely. In this paper, we split the running path [x_{start}, x_{end}] into different length intervals according to the limited speed V as well as the line conditions, *e.g.* grade profiles, tunnels, curves and neutral sections. The interval length can be described as $\Delta x_k = x_{k+1}-x_k$, for $k = 1,2,\ldots,N$. Note that

$$\forall\, S_i,\ i\in(1,2,\ldots,K),\ \exists\, x_k,\, k\in(1,2,\ldots,N)\ \Rightarrow\ S_i = x_k$$

The objective function can be rewritten as:

$$\min\ J_r = \sum_{k=1}^{N}\left(\frac{F(k)}{\eta} - \alpha B_E(k)\right)\Delta x_k \tag{10}$$

where $F(k)$ represents the traction force in interval [x_k, x_{k+1}], $B_E(k)$ represents the regenerative braking force in interval [x_k, x_{k+1}], and both of them are constant in each interval.

The dynamics equation of train motion (2) can be rewritten as [10]:

$$\begin{cases} E(k+1) = a_k E(k) + b_k(F(k) - B_A(k) - B_E(k)) + c_k \\ t(k+1) = t(k) + \dfrac{1}{2}\left(\dfrac{1}{\sqrt{2E(k)}} + \dfrac{1}{\sqrt{2E(k+1)}}\right)\Delta x_k \end{cases} \tag{11}$$

where the coefficients a_k, b_k and c_k are defined appropriately [10], $E(k)$ represents the kinetic energy per mass in position x_k, $B_A(k)$ represents the constant air braking force in interval [x_k, x_{k+1}], $t(k)$ represents the running time in position x_k. Note that $t(1) = 0$, $t(N+1) = T$ and $E(1) = E(N+1) = 0$.

3.2 The PWA Approximation

The time equation of the differential equation (11) contains the nonlinear function:

$$f(E(k)) = \frac{1}{2\sqrt{2E(k)}}$$

which cannot be formulated in the MILP model. Thus, in this paper, the nonlinear function $f(E)$ is approximated by PWA method, which can be transformed to a piecewise linear function. We consider an approximation using 2 affine subfunctions, which can be achieved by MATLAB toolbox. Then, the nonlinear function $f(E)$ can be transformed to:

$$f_{\text{PWA}}(E) = \begin{cases} \alpha_{1,k}E + \beta_{1,k} & E_{0,k} \le E \le E_{1,k} \\ \alpha_{2,k}E + \beta_{2,k} & E_{1,k} \le E \le E_{2,k} \end{cases} \tag{12}$$

where $E_{0,k}$ represents the minimum kinetic energy per mass in position x_k, $E_{2,k}$ represents the maximum kinetic energy per mass in position x_k, $E_{1,k}$ represents a parameter generated in approximation, the coefficients $\alpha_{1,k}$, $\beta_{1,k}$, $\alpha_{2,k}$ and $\beta_{2,k}$ can be generated in approximation.

According to the piecewise linear function (12), the time equation of the differential equation (11) can be written as:

$$t(k+1) = t(k) + (\alpha_l E(k) + \beta_l + \alpha_m E(k+1) + \beta_m)\Delta x_k \tag{13}$$

where l, m = 1or 2, depended on the value of $E(k)$ and $E(k+1)$ respectively.

3.3 The Mixed Logical Dynamic Model

Although, the nonlinear function has been approximated to a piecewise function, the logical judgment cannot be formulated in MILP model. Thus, we introduce logical variables $\delta(k)$ to describe the logical conditions, defined as:

$$\begin{cases} [E(k) \ge E_{1,k}] \Leftrightarrow [\delta(\mathrm{k}) = 1] \\ [E(k) < E_{1,k}] \Leftrightarrow [\delta(k) = 0] \end{cases}$$

Considering the value range [$E_{0,k}$, $E_{2,k}$] of $E(k)$, the logical conditions can be rewritten as:

$$\begin{cases} (E_{1,k} - E_{2,k} - \varepsilon)\delta(k) \le E_{1,k} - E(k) - \varepsilon \\ (E_{1,k} - E_{0,k})\delta(k) \le E(k) - E_{0,k} \end{cases} \tag{14}$$

where ε is a small positive number to transform a strict inequality into an inequality, which fits the MILP model framework [10].

Additionally, we introduce auxiliary variables $z(k) = \delta(k)E(k)$, which can be expressed as [10]:

$$\begin{cases} z(k) \le E_{2,k}\delta(k) \\ z(k) \ge E_{0,k}\delta(k) \\ z(k) \le E(k) - E_{0,k}(1-\delta(k)) \\ z(k) \ge E(k) - E_{2,k}(1-\delta(k)) \end{cases} \tag{15}$$

Based on the variables $\delta(k)$ and $z(k)$, the expression $\alpha_l E(k)+\beta_l$ can be transformed to a linear equation as:

$$(\alpha_{2,k} - \alpha_{1,k})z(k) + (\beta_{2,k} - \beta_{1,k})\delta(k) + \alpha_{1,k}E(k) + \beta_{1,k}$$

thus, the time equation (13) can be transformed to a linear equation.

Similarly, the varying maximum traction force can be reformed as a nonlinear function of the kinetic energy:

$$F_{\max,\text{PWA}} = \begin{cases} \mu_1 & E_{0,k} \le E \le E_{3,k} \\ \lambda E + \mu_2 & E_{3,k} \le E \le E_{2,k} \end{cases} \tag{16}$$

where the parameters μ_1, μ_2 and λ are simplified to be same in each interval, the varying maximum braking force B_E and B_A can be formulated as the from above.

Then, the dynamical equation of state variables E and t can be described as the following mixed logical dynamic model:

$$\begin{aligned} s(k+1) = {} & G_{1,k}s(k) + G_{2,k}(F(k) - B_E(k) - B_A(k)) \\ & + G_{3,k}\delta(k) + G_{4,k}\delta(k+1) + G_{5,k}z(k) \\ & + G_{6,k}z(k+1) + G_{7,k} \end{aligned} \tag{17}$$

where $s(k) = [E(k)\ t(k)]^T$, $G_{1,k} = \begin{bmatrix} a_k & 0 \\ \Delta x_k(\alpha_{1,k} + a_k\alpha_{1,k+1}) & 1 \end{bmatrix}$, and $G_{2,k}$, $G_{3,k}$, $G_{4,k}$, $G_{5,k}$, $G_{6,k}$ and $G_{7,k}$ are defined in the similar way as $G_{1,k}$.

Meanwhile, the constrains (6), (7), (9), (14) (15) and(16) can be rewritten more compactly as:

$$\begin{aligned} R_{1,k}(F(k) - B_E(k) - B_A(k)) + R_4 s(k) \\ + R_{2,k}\delta(k) + R_{3,k}z(k) \le R_{5,k} \end{aligned} \tag{18}$$

where the coefficient matrixes $R_{i,k}$, for i = 1,2,…,5, are defined appropriately.

3.4 The MILP Model

Based on the transformation above, the EETC problem can be rebuilt as a MILP model [11]:

$$\min \; C\psi \tag{19}$$

subject to

$$F_{11}\psi \le F_{12}s(1) + F_{13} \tag{20}$$

where $C = [0 \ldots 0 \;\; \frac{1}{\eta}\Delta x_1 \ldots \frac{1}{\eta}\Delta x_N \;\; -\alpha\Delta x_1 \ldots -\alpha\Delta x_N \;\; 0 \ldots 0]$, $\psi = [\delta' \; z' \; F' \; B_E' \; B_A']^T$, $\delta' = [\delta(1) \; \delta(2) \ldots \delta(N+1)]$ and z', F', B_E' and B_A' are defined in the similar way as δ'. The coefficient matrixes F_{11}, F_{12} and F_{13} are defined appropriately. The state variable $s(k)$ can be expressed by $s(1)$ as [10]:

$$\begin{aligned} s(k) = {} & \left[\prod_{j=1}^{k-1} G_{1,j}\right] s(1) + \sum_{i=1}^{k-1}\left[\prod_{j=i+1}^{k-1} G_{1,j}\right] G_{2,i}(F(k) - B_E(k) \\ & - B_A(k)) + \left[\prod_{j=2}^{k-1} G_{1,j}\right] G_{3,1}\delta(1) + \sum_{i=2}^{k-1}\left[\prod_{j=i+1}^{k-1} G_{1,j}\right](G_{1,i}G_{4,i-1} \\ & + G_{3,i})\delta(i) + G_{4,k-1}\delta(\mathrm{k}) + \left[\prod_{j=2}^{k-1} G_{1,j}\right] G_{5,1}z(1) + \sum_{i=2}^{k-1}\left[\prod_{j=i+1}^{k-1} G_{1,j}\right] \\ & (G_{1,i}G_{6,i-1} + G_{5,i})z(i) + G_{6,k-1}\,\mathrm{z(k)} + \sum_{i=1}^{k-1}\left[\prod_{j=i+1}^{k-1} G_{1,j}\right] G_{7,i} \end{aligned}$$

The MILP problem above can be solved by CPLEX solvers in MATLAB efficiently.

4 State-space Improvement

4.1 State-spaces of Train Control Problem

In this paper, we choose the kinetic energy per mass unit E and the running time t as state variables, the running position x as the independent variable. Additionally, the time equation (13) is a function of state variable E, which indicates that constrains of the running time affect the valuating of state variable E substantially. Meanwhile, the state variable E can be described by v, so the essentiality of EETC problem is to search optimal train track profiles in state-spaces like Fig. 1.

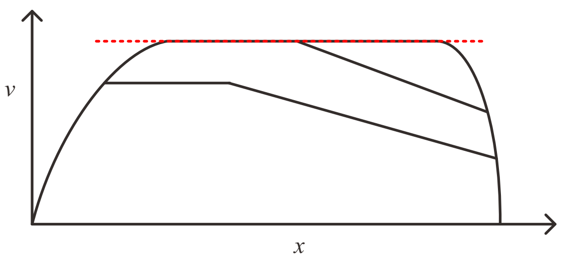

Fig. 1: Optimal train track profiles with different travel times

With regard to the discrete-position model, the state variable E is discretized to N+1 parts corresponding to each interval. Thus, the feasible values of $E(k)$, for k = 1,2,…N+1, constitute the state-spaces of EETC problem discussed in this paper.

4.2 Strategy to Exclude Non-optimal Region

The computation time is positive related to spacing of admissible state-spaces. Therefore, it is essential to confine state-spaces, divide into non-optimal region and admissible region, in order to shorten the computation time. Ko et. al [7] divide the state-spaces into qualitatively non-optimal region and admissible region depend on the total travel time. The non-optimal region is determined without quantitative analysis. In this paper, we propose several strategies to exclude non-optimal region as following:

1) The maximum of state variable E is bounded by the maximum speed V, so the optimal track profile cannot reach Region 1 as shown in Fig. 2.

2) The maximum acceleration in starting stage is bounded by the maximum traction force $F_{\max}$, so the optimal track profile cannot reach Region 2.

3) The maximum deceleration in parking stage is bounded by the maximum barking force $B_{E,\max}$ or $B_{A,\max}$, so the optimal track profile cannot reach Region 3.

4) The optimal driving strategy is proven to consist of maximum acceleration in starting stage [5], so the optimal track profile is avoided getting close to Region 4.

5) The optimal driving strategy is proven to consist of maximum deceleration in parking stage [5], so the optimal track profile is avoided getting close to Region 5.

6) In order to keep punctuality, the running speed in running stage (cruising or coasting) should be higher than the average running speed V_a, defined as $V_a = (x_{\mathrm{end}} - x_{\mathrm{start}})/T$. Meanwhile, drastic speed fluctuation will largen the energy consumption [5], so the optimal track profiles is avoided getting close to Region 6.

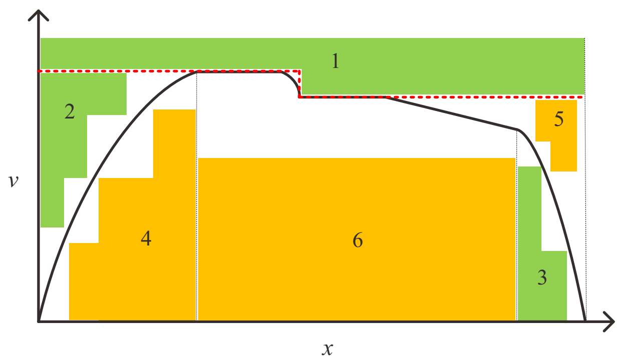

Fig. 2: Confined state-space and non-optimal region

To sum up, Region 1, 2 and 3 constitute the unreachable state-spaces, which are described in constrains (6), Region 4, 5 and 6 constitute the non-optimal region, the rest of state-spaces is admissible region. According to definitions of logical variable δ and auxiliary variables z, we define Region 4 and 6 as:

$$\delta(k) = 1, \;\; z(k) = E(k) \le E_a(k)$$

and Region 5 as:

$$\delta(k) = 1, \;\; z(k) = E(k) \ge E_a(k)$$

where the value range of k is based on the phase of driving strategy, E_a is determined by the non-optimal region limit. Note that, the value range of k is based on the experiential train track profiles. E_a in Region 4 and 5 are determined by the characteristic of train traction and breaking force, and E_a in Region 6 is determined by V_a.

The region definitions can be described as:

$$F_{21}\psi \le F_{22}s(1) + F_{23} \tag{21}$$

where the coefficient matrixes F_{21}, F_{22} and F_{23} are defined appropriately. Then, the state-spaces improvement (SSI) can be formulated in MILP model.

5 Case Studies

In this section, we show two case studies, one between two stations, another one between four consecutive stations. The specifications of the train (CRH3C high-speed train) used in case studies are presented in Table 1 and Fig. 3.

Table 1: The basic parameters of train

Property	Symbol	Value
Organization		4M4T
Train mass [t]	m	475
Rotating mass factor	ρ	1.06
Maximum speed [km/h]	V	350
Efficiency of traction	η	0.9
Efficiency of regenerative braking	α	0.65

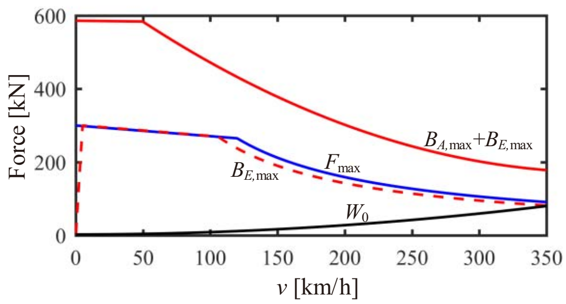

Fig. 3: Traction/breaking performances and running resistance

5.1 Simulation between two stations

The simulation is based on the line from Hengyangdong station to Leiyangxi station, parts of the Wuguang high-speed railway line in China. The length of the simulated line equals to 54.626 km, and the total travel time equals to 16 minutes. The line contains 2 neutral sections in intervals [9.652, 10.852] and [34.216, 25.416]. We build the simulation based on strategies excluding non-optimal state-spaces, comparing with the simulation without state-spaces improvement (SSI). The results are shown in Fig. 4 and Table 2. The running section is divided into 55 intervals, and $E_a(k) = 868$, for $k = 5,\dots,10$; $E_a(k) = 1400$, for $k = 10,\dots,45$; $E_a(k) = 1867$, for $k = 50,\dots,55$.

The simulation results show that the computation cost of the MILP model is very low, the energy-efficient control strategy can be obtained less than 10 seconds. Meanwhile, leaning state-spaces by using strategies mentioned in section 4.2 can reduces computation time effectively, comparing Simulation with SSI and Simulation without SSI.

Table 2: The solver performances of two simulations

Property	Simulation with SSI	Simulation without SSI
J_r [kWh]	860.7	860.7
computation time [s]	0.313	8.140
computation iterations	213	225503
time error [s]	1.0	1.0

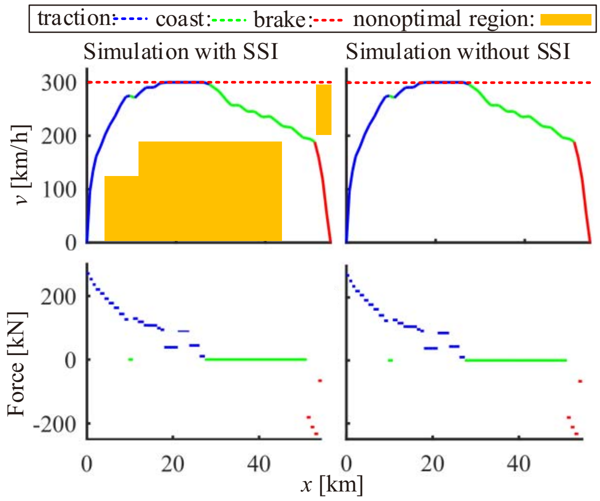

Fig. 4: The optimal track profiles and the input

5.2 Simulation of consecutive stations

In this case study, we apply the improved MILP model to a more complicated problem, searching an EETT and its corresponding control strategy between four consecutive stations. The simulation is based on the line form Miluodong station to Leiyangxi station with two middle stations. The lengths of each running section are 76.805, 160.171 and 54.626 km. The optimal train track profiles for original and optimal timetables are shown in Fig. 5, and the simulation results are shown in Table 3.

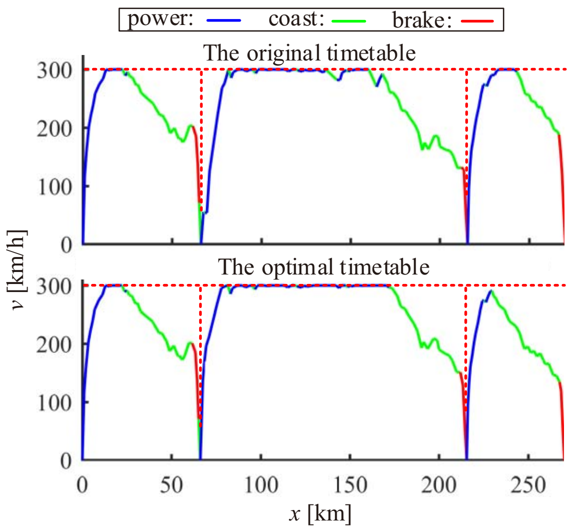

Fig. 5: The optimal track profiles of original and optimal timetables

The simulation results show that optimizing timetable can reduce the total energy consumption of the running path, by adjusting the running time of each section under the total travel time constrains. The optimal control strategy under the optimal timetable can reduce 0.67% energy consumption compared to the one under the original timetable, the energy efficient will be better compared to the non-optimal control strategy under the original timetable. With the superiority of the improved MILP model, the computation time is less than 1 minute.

Table 3: The simulation results of original and optimal timetables

Property		A-B	B-C	C-D	Total
J_r [kWh]	original	811.1	2173.4	860.7	3845.2
	optimal	797.6	2374.0	648.0	3819.6
	variation	−13.5	+200.6	−212.7	-25.6
T [s]	original	1260	2220	961	4441
	optimal	1279	1911	1251	4441
	variation	+19	−309	+290	0

6 Conclusion

In this paper, the EETT problem considering EETC in both single trajectory and multiple consecutive trajectories have been discussed. A MILP model is built via non-uniform discretization according to line conditions, *e.g*. grade profile, tunnels and curves. Meanwhile, the regenerative breaking and neutral section constraints have been taken into account in this model. Moreover, this paper peoposes a strategy to exclude the non-optimal state-spaces, in order to improve the computational efficiency of MILP solver. Furthermore, the state-space improvement makes the EETT solving process more efficiently, and the results show that the optimizing timetable can reduce the energy consumption under the total travel time constrain.

In future studies, more operational constraints and realistic conditions will be added into the proposed MILP model, *e.g*. the variable utilization rate of regenerative energy, the influence of the signal system and the limit of equipment resources. Meanwhile, the optimization of multi-train system will be considered.

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